Principles of Robot Autonomy II

Model-based and Model-free RL for Robot Control

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Autonomous Systems Lab

Learning from Experience

How to use trajectory data?

- Model based approach: estimate $T(x'|x, u)$, then use model to plan
- Model free:
  - Value based approach: estimate optimal value (or Q) function from data
  - Policy based approach: use data to determine how to improve policy
  - Actor Critic approach: learn both a policy and a value/Q function

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Model-free, policy based: Policy Gradient

Alternative: instead of learning the Q function, learn the policy directly!

Define a class of policies π_θ where θ are the parameters of the policy

Can we learn the optimal θ from interaction?

Goal: use trajectories to estimate a gradient of policy performance w.r.t. parameters θ

Policy Gradient

A particular value of θ induces a distribution $p(\tau; \theta)$ over possible trajectories

- Distribution comes from stochastic dynamics $T(x' | x, u)$ as well as stochastic policy $u \sim \pi(\cdot \mid x; \theta)$.

Objective function:

$$J(\theta) = E_{\tau \sim p(\tau;\theta)}[r(\tau)]$$

i.e.,

$$J(\theta) = \int_{\tau} r(\tau) p(\tau; \theta) d\tau$$

where $r(\tau)$ is the total discounted cumulative reward of a trajectory τ

Policy Gradient

Gradient of objective w.r.t. parameters:

$$\nabla_\theta J(\theta) = \int_\tau r(\tau)\nabla_\theta p(\tau;\theta)d\tau$$

Trick: $\nabla_\theta p(\tau;\theta) = p(\tau;\theta)\frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta)\nabla_\theta \log p(\tau;\theta)$

$$\nabla_\theta J(\theta) = \int_\tau (r(\tau)\nabla_\theta \log p(\tau;\theta))p(\tau;\theta)\, d\tau$$

$$\nabla_\theta J(\theta) = E_{\tau\sim p(\tau;\,\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

Policy Gradient

$$\nabla_\theta J(\theta) = E_{\tau \sim p(\tau;\,\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

$$\log p(\tau;\theta) = \log\left(\prod_{t\geq 0} T(x_{t+1}|x_t,u_t)\pi_\theta(u_t|x_t)\right)$$

$$= \sum_{t\geq 0} \log T(x_{t+1}|x_t,u_t) + \log \pi_\theta(u_t|x_t)$$

$$\Rightarrow \ \nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(u_t|x_t)$$

Policy Gradient

$$\nabla_\theta J(\theta) = E_{\tau \sim p(\tau;\, \theta)}[r(\tau) \nabla_\theta \log p(\tau; \theta)]$$

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$$\Rightarrow\ \nabla_\theta \log p(\tau; \theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(u_t|x_t)$$

We don't need to know the transition model to compute this gradient!

Policy Gradient

If we use π_θ to sample a trajectory, we can approximate the gradient via N Monte Carlo samples:

$$\nabla_\theta J(\theta) = E_{\tau\sim p(\tau;\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

$$\approx \frac{1}{N}\sum_{i=1}^{N}\left(r\left(\tau^{(i)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta(u_t^{(i)}|x_t^{(i)})\right)$$

Intuition: adjust θ to:

- Boost probability of actions taken if reward is high
- Lower probability of actions taken if reward is low

Learning by trial and error

Time dependency of policy gradient theorem

- Previous estimator for policy gradient was

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(r\left(\tau^{(i)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta(u_t^{(i)}|x_t^{(i)})\right)$$

Action $u_{t'}$ can not change reward r_t for $t < t'$ (i.e., previous timesteps):

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t\geq 0}\nabla_\theta \log \pi_\theta(u_t^{(i)}|x_t^{(i)})\sum_{k\geq t} r(x_k^{(i)}, u_k^{(i)})\right)$$

(caveat: this is not a rigorous argument we’re presenting here)

REINFORCE

Loop forever:

Generate episode $x_0, u_0, r_0, x_1, u_1, r_1$ … with π_θ

Loop for all $t = 0, \dots, N-1$:

$G_t \leftarrow \sum_{k=t}^{N} r_k$

$\theta \leftarrow \theta + \alpha\, G_t\, \nabla_\theta \log \pi_\theta(u_t | x_t)$

Cumulative tail reward, the tail "return"

Policy Gradient Recap

Pros:

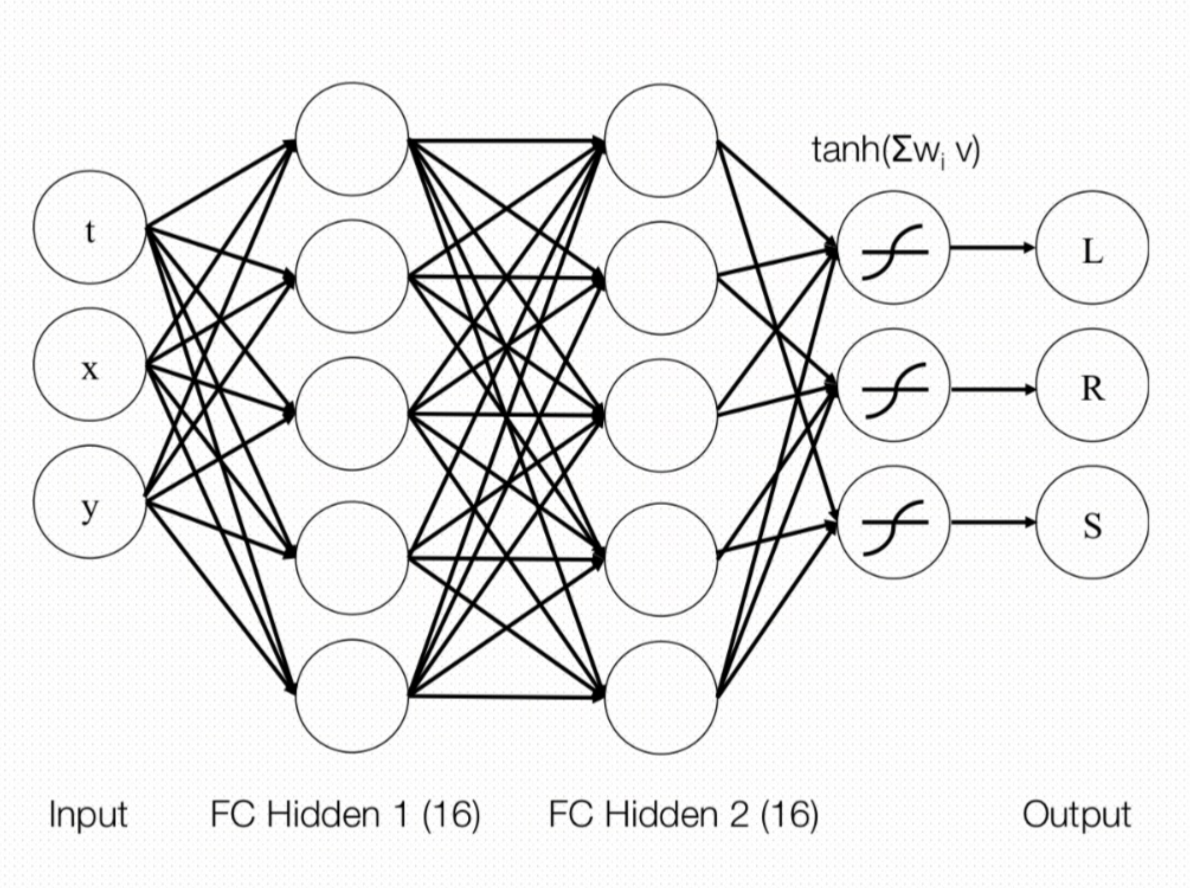
- Learns policy directly – can be more stable (less moving parts than *Q*-learning*)*
- Works for continuous action spaces (no need to "argmax" *Q*)
- Converges to local maximum of $J(\theta)$

Cons:

- Needs data from current policy to compute gradient – data inefficient
- Gradient estimates can be very noisy
  - Need to reduce variance of gradient estimator: baselines and actor-critic

Deep Reinforcement Learning

- Deep Q learning:
  - Use neural network as Q function
  - Works in continuous state space domains
- Deep Policy Gradient:
  - Parameterize policy as deep neural network
  - Policy can act on high dimensional input, e.g., directly from visual feedback



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Tabular model-based RL

- Discrete state/action space with stochastic transitions
- If model is known, can use value iteration/policy iteration/etc.
- Model unknown: want to build approximate model from observed transitions

Tabular MBRL outline

- Assume initial policy
- Loop forever:
  - Take some number of actions, resulting in transition/reward data
  - Improve dynamics model
  - Choose actions/policy

- Approaches for action selection:
  - Dynamic programming/VI/etc. on approximate model
    - Expensive, gives optimal policy for model
  - Plan suboptimal sequence of actions via online control optimization

Learning a tabular model from data

- States $(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n)$
- Actions $(\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_m)$
- Want to learn $p(\mathbf{x}_i | \mathbf{x}_\mathrm{j}, \mathbf{u}_k)$ for all i, j, k

- Main strategies:
  - max likelihood point estimation
  - Bayesian approaches

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Max likelihood for tabular MBRL

- Categorical likelihood: $p(\mathbf{x}_i | \mathbf{x}_j, \mathbf{u}_k, \boldsymbol{\theta}) = \boldsymbol{\theta}_{ijk}; \sum_i \boldsymbol{\theta}_{ijk} = 1$
- Assume data $D = \{(\mathbf{x}, \mathbf{u}, \mathbf{x}')\}_{i=1}^{d}$
- Max likelihood:

$$\max_{\theta \in \Theta} \sum_{D} \log p(\mathbf{x}' | \mathbf{x}, \mathbf{u}, \boldsymbol{\theta})$$

- Optimizing this gives the maximum likelihood estimate

$$\widehat{\boldsymbol{\theta}}_{ijk} = \frac{N(\mathbf{x}_j, \mathbf{u}_k, \mathbf{x}_i)}{N(\mathbf{x}_j, \mathbf{u}_k)}$$

where $N(\cdot,\cdot)$ is the empirical count

Max likelihood for tabular MBRL

- $\boldsymbol{\theta}_{ijk} = N(\mathbf{x}_j, \mathbf{u}_k, \mathbf{x}_i)/N(\mathbf{x}_j, \mathbf{u}_k)$
- Problem: what if $N(\mathbf{x}_j, \mathbf{u}_k) = 0$?
  - For example, if we are starting with zero information, this model estimation scheme breaks
- Simple solution: start all of our counts at 1, i.e.,
  - Store $N(\mathbf{x}_j, \mathbf{u}_k, \mathbf{x}_i)$; note that $N(\mathbf{x}_j, \mathbf{u}_k) = \sum_{\mathbf{x}_i} N(\mathbf{x}_j, \mathbf{u}_k, \mathbf{x}_i)$
  - Replace $N(\mathbf{x}_j, \mathbf{u}_k, \mathbf{x}_i)$ with $N(\mathbf{x}_j, \mathbf{u}_k, \mathbf{x}_i) + 1$
  - Gives $\boldsymbol{\theta}_{ijk} = (N(\mathbf{x}_j, \mathbf{u}_k, \mathbf{x}_i) + 1)/(N(\mathbf{x}_j, \mathbf{u}_k) + n)$

Why model-based?

- Advantages
  - Transitions give strong signal
  - Data efficiency, improved multi-task performance, generalization

- Weaknesses
  - Optimizing the wrong objective (i.e., not your ultimate task of optimizing reward)
  - May be very difficult/intractable for systems with high dimensional observations/states

Challenges in RL for Robotics

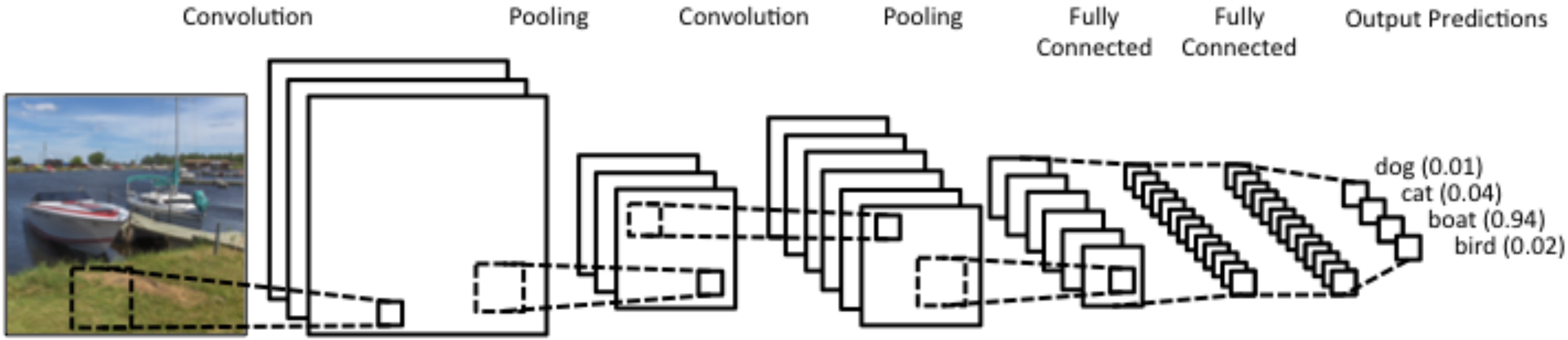
Data-efficiency

Sim-to-real

Exploration

Reward design

Next time


Convolution
Pooling
Convolution
Pooling
Fully Connected
Fully Connected
Output Predictions
dog (0.01)
cat (0.04)
boat (0.94)
bird (0.02)